



TRIMBLE ACCESS™ CALCULATIONS

Revision A
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Calculations performed by the Trimble Access software

This document outlines some of the calculations performed by the Trimble Access software. These calculations fall into the following groups:

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Transformations applied to GNSS positions

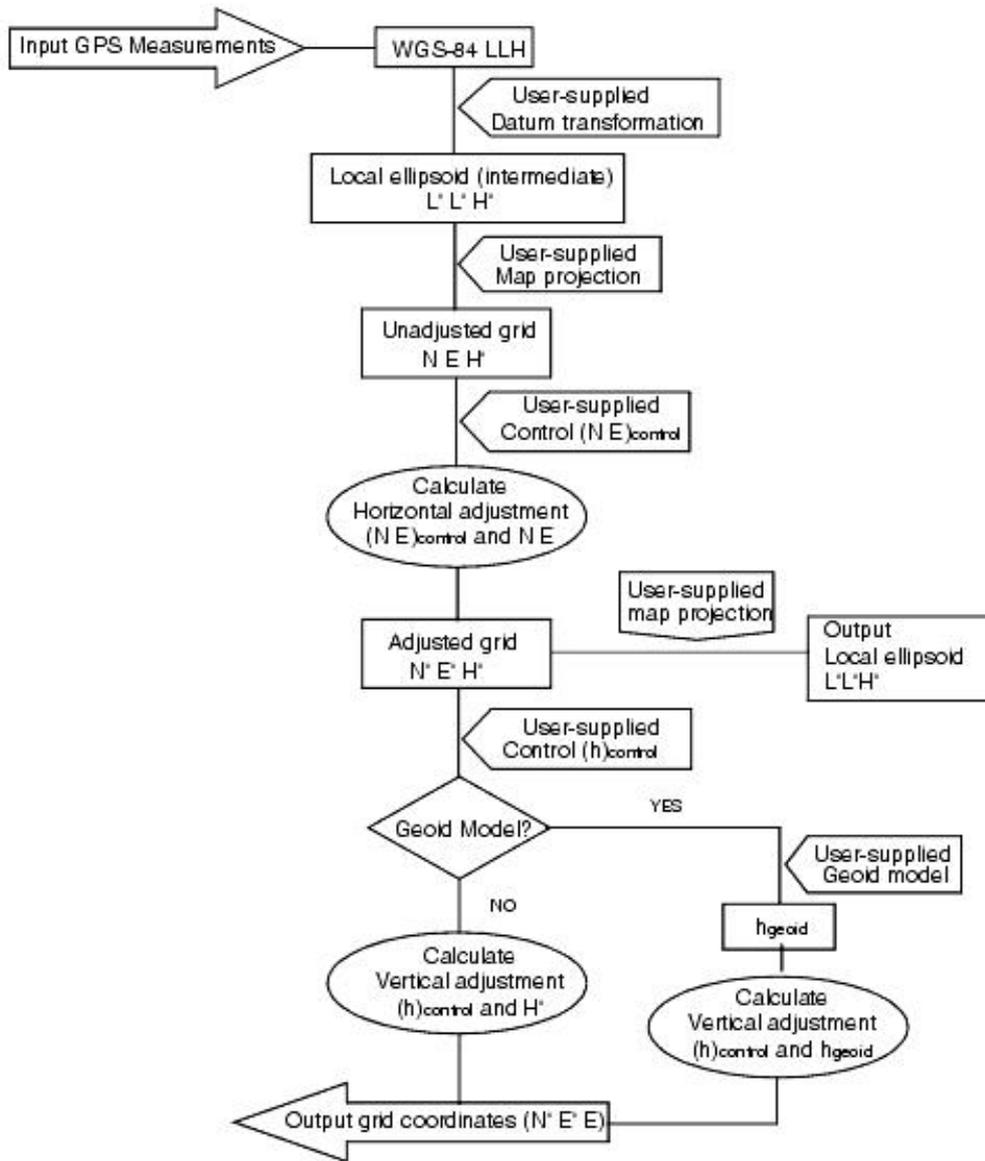
For RTK surveying, coordinate transformations are required so that one set of coordinates (GNSS positions) can be represented in terms of another (grid positions), and vice versa.

This section provides an overview of the management and application of coordinate transformations using the Trimble Access software. It describes how to apply a datum transformation and map projection, and horizontal and vertical adjustments.

With the Trimble Access software, the site calibration process defines transformation parameters derived from a set of points. This set of points is coordinated in terms of two systems:

- WGS-84 Latitude, Longitude, Height (LLH) geodetic coordinates
- A local system with project-specific Northing, Easting, Elevation (NEE) grid coordinates

The following diagram shows the order of calculations performed when a calibration is calculated. The formulae used in the diagram are detailed below.



Transforming WGS-84 ECEF to WGS-84 LLH

When GNSS signals are processed by a receiver, they yield Earth-Centered-Earth-Fixed (X, Y, Z) coordinates, which must then be transformed to more meaningful geodetic coordinates (φ , λ , H).

Here, φ represents the geodetic latitude, λ is the longitude, and H is the perpendicular height above the WGS-84 ellipsoid.

First we define:

$$N = \frac{r}{\sqrt{1 - e^2 \sin^2(\varphi)}}$$

$$e^2 = 2\varphi - \varphi^2$$

where φ is the flattening value for the source ellipse and r is the semi-major axis.

The values of the ECEF coordinates are:

$$X = (N + H) \cdot \cos(\varphi) \cdot \cos(\lambda)$$

$$Y = (N + H) \cdot \cos(\varphi) \cdot \sin(\lambda)$$

$$Z = [N(1 - e^2) + H] \cdot \sin(\varphi)$$

The inverse problem (that of transforming ECEF coordinates to φ , λ , and H) is solved by using an iterative procedure. The values of e^2 and N now use the destination ellipsoid flattening and semi-major axis values:

$$\varphi = \tan^{-1} \left(\frac{Z}{\sqrt{X^2 + Y^2}} (1 - e^2) \right)$$

then iterate

$$\varphi = \tan^{-1} \left(\frac{Z + e^2 N \sin(\varphi)}{\sqrt{X^2 + Y^2}} \right)$$

$$\lambda = \tan^{-1} \left(\frac{Y}{X} \right)$$

if $45^\circ \text{S} < \varphi < 45^\circ \text{N}$

$$H = \frac{\sqrt{X^2 - Y^2}}{\cos(\varphi)} - N$$

or if $\varphi > 45^\circ \text{N}$ or $\varphi < 45^\circ \text{S}$

$$H = \frac{Z}{\sin(\varphi)} - N(1 + e^2)$$

Datum transformation

A datum transformation provides the necessary parameters for converting from one geodetic coordinate system to another.

The Trimble Access software can apply a predefined three- or seven-parameter datum transformation. It can also calculate a three-parameter datum transformation given points coordinated in WGS-84 and local L'L'H'.

$$\mathbf{X} = \mathbf{T} + \mathbf{kRX}'$$

where \mathbf{X}' is a matrix of 3-D Cartesian ECEF coordinates or Cartesian local coordinates, \mathbf{T} is a matrix of translation parameters, \mathbf{k} is a scalar, and \mathbf{R} is a rotation matrix. In most cases, \mathbf{X}' is measured and \mathbf{T} , \mathbf{k} , and \mathbf{R} are user-specified.

To calculate a three-parameter datum transformation, pairs of WGS-84 LLH and local L'L'H' coordinates are needed.

In the trivial one point case, the three translation parameters are just the vector components of the ECEF vector that connect the ECEF pair derived from the WGS-84 LLH and the local L'L'H'.

In the non-trivial case, the translation parameters are the vector components of the average vector. This is represented as:

$$\mathbf{AX} + \mathbf{W} = \mathbf{0}$$

where the solution

$$\mathbf{X} = \begin{bmatrix} \Delta X \\ \Delta Y \\ \Delta Z \end{bmatrix}$$

and

$$\mathbf{W} = \begin{bmatrix} X_1 - X'_1 \\ Y_1 - Y'_1 \\ Z_1 - Z'_1 \\ X_2 - X'_2 \\ Y_2 - Y'_2 \\ Z_2 - Z'_2 \\ \vdots \end{bmatrix}$$

where \mathbf{X}_n is the value of the ECEF coordinate derived from the local L'L'H' of the nth 3-D point in the list, and \mathbf{X}'_n is the \mathbf{X} value of the ECEF coordinate derived from the WGS-84 LLH of the nth 3-D point and

$$\mathbf{A} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ \vdots \end{bmatrix}$$

is called the Molodensky matrix.

For more information, refer to *GPS Satellite Surveying* (A. Leick; John Wiley & Sons, 1995).

Map projection

A map projection defines the relationship between the local ellipsoidal surface (L'L'H') and a plane. Generally, map projection parameters are based on a local conformal mapping model.

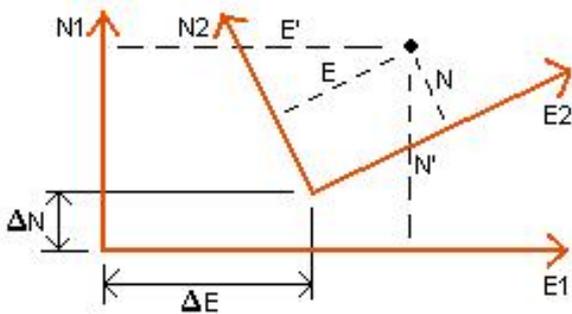
For more information about map projections, please refer to *Map Projections--A Working Manual* (J.P. Snyder; U.S. Geological Survey Professional Paper 1295, U.S. Government Printing Office, Washington, 1987).

For nearly all projection types used in the Trimble Access software, the height component of a projected coordinate is just the height above or below the datum at that point. For a plane projection, however, the definition includes an ellipsoidal height and a projected surface elevation at the origin point.

Horizontal adjustment

It may be necessary to minimize the discrepancy between local fixed control coordinates (NEcontrol) and projected grid coordinates (N'E'). The horizontal adjustment solves parameter translations in north and east (ΔN , ΔE), a rotation φ , and a scale factor k using two different sets of plane coordinates--one converted from measurements in the field, and the other from a control list. Where the Trimble Access software generates its own three-parameter datum transformation, it is necessary to provide for scaling and rotation. This is done by a horizontal adjustment.

The following diagram shows the transformation between two coordinate systems.



Coordinate systems for a horizontal adjustment

The Trimble Access software minimizes the discrepancy between local NE control and NE values derived using GNSS observations and a datum transformation and map projection. It does this by performing a plane horizontal least-squares adjustment with no weighting.

In the trivial one point case, the translation parameters are just the Northing and Easting components of the vector between the two coordinated values. The scale factor is one, and the rotation value is zero.

For two or more points, calculation of horizontal adjustment uses a simple four-parameter transformation. This solves for two translations (ΔN , ΔE), a rotation (φ), and a scale factor (k) between coordinate pairs.

The geometry between the two coordinate systems results in two transformation equations:

$$N' = aN + bE + \Delta N$$

$$E' = -bN + aE + \Delta E$$

where $a = k \cos \varphi$ and $b = k \sin \varphi$ are used to simplify the matrix representation, and ΔN and ΔE represent the shifts of the N and E axis in the N' and E' system.

Common points in both coordinate systems are used in a least-squares adjustment to solve the four unknown parameters (a , b , ΔE , and ΔN).

Once estimates of a and b are determined, the rotation and scale between the two systems is calculated by:

$$\varphi = \tan^{-1} \left(\frac{a}{b} \right) \text{ and } k = \sqrt{a^2 + b^2}$$

For more information about horizontal adjustment, please refer to *Observations and Least Squares* (E. Mikhail: John Wiley & Sons, 1982).

Vertical adjustment

The Trimble Access software determines a vertical adjustment using least-squares with no weighting. This adjustment requires measured WGS-84 heights and control elevations.

In the trivial one-point case, the adjustment consists of a constant height shift only. For two or more points, a tilt in north and east are also computed.

The inclined plane parameters are determined by solving the matrix equation:

$$\mathbf{AX} = \mathbf{B}$$

where the solution

$$\mathbf{X} = \begin{bmatrix} \Delta H \\ \Delta E \\ \Delta N \end{bmatrix}$$

the components being the constant height shift and tilt east and north (in terms of a height shift per unit distance east or north), and the design matrix

$$\mathbf{A} = \begin{bmatrix} 1 & E_1 - E_1 & N_1 - N_1 \\ 1 & E_2 - E_1 & N_2 - N_1 \\ & \vdots & \\ 1 & E_n - E_1 & N_n - N_1 \end{bmatrix}$$

where E_n N_n are the coordinates of the nth point as derived from the WGS-84 data set.

E_1 N_1 are the coordinates of the origin point of the adjustment. (The origin point can be any one of the n points.)

$$\mathbf{B} = \begin{bmatrix} H'_1 - H_1 \\ H'_2 - H_2 \\ \vdots \\ H'_n - H_n \end{bmatrix}$$

where $H'_n - H_n$ is the difference in elevation between the keyed in value for the nth point and the value derived from the WGS-84 data set.

Ground scale factor

In the Trimble Access software, you can define a *Project height* when you create a *No projection/no datum* job. The project height is used after you perform a site calibration to compute a scale factor for the projection, so that ground coordinates are computed at elevation.

The projection scale factor is calculated as follows:

$$SF = \frac{R + h}{R}$$

and

$$R = \frac{a}{\sqrt{1 - e^2 \sin^2(\varphi)}}$$

where:

<i>SF</i>	scale factor of projection
<i>R</i>	ellipsoidal radius
<i>h</i>	average elevation (reference elevation)
<i>a</i>	semi major axis
e^2	eccentricity ²
<i>φ</i>	origin latitude of projection

Geoid model

The Trimble Access software can use a geoid model to provide orthometric heights from measured (by GNSS) WGS-84 heights.

Geoid model is one of the options in the *Vertical adjustment* field. (The other options in this field are *No adjustment*, *Inclined plane*, *Geoid model/Inclined plane*).

If you select Geoid model and you do not perform an in-field calibration, the elevation values displayed by the Trimble Access software represent the unadjusted elevation above the defined geoid using the relationship:

$$h_{\text{geoid}} = H - N$$

where:

<i>h_{geoid}</i>	the unadjusted elevation above the geoid
<i>H</i>	the measured GNSS height above the ellipsoid
<i>N</i>	the geoid ellipsoidal separation, derived from a geoid model

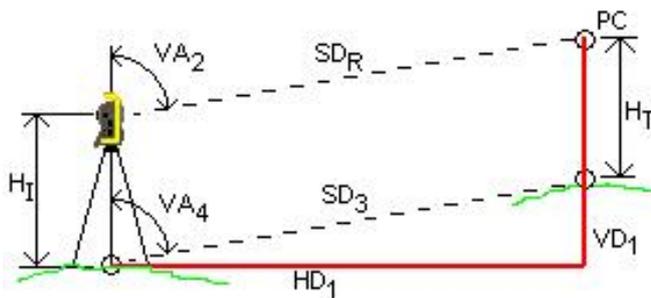
If you select *Geoid model* and then perform an in-field calibration, the Trimble Access software computes the calibration parameters using *hcontrol* and *hgeoid* as inputs, so that the geoid model is inclined to fit the local control elevations. The method of vertical adjustment becomes *Geoid/Inclined plane*.

Ellipsoid calculations

The ground and ellipsoid distances in the Trimble Access software are calculated parallel to the ellipsoid. The equations used for these calculations are based on the Robbins ellipsoid geometry formulas. These formulas (by Dr A.R. Robbins) are in the *Empire Survey Review* No. 125, 1962. They are accurate to better than 20 mm over distances of 1,500 km. The errors can reach 16 meters at 4,500 km and more than 2,000 meters at 9,000 km.

Conventional instrument calculations

The following diagram labels the observations and corrections that are applied when you use the Trimble Access software with a conventional instrument.



Where:

- VA_2** Vertical angle from conventional instrument.
The Trimble Access software assumes that the conventional instrument applies any corrections for collimation and tilt.
- $[VA_3]$** Vertical angle corrected for curvature and refraction
- VA_4** Vertical angle corrected for curvature and refraction, instrument and target heights
- SD_R** Slope distance from EDM
- $[SD_1]$** Slope distance corrected for prism constant (PC)
- $[SD_2]$** Slope distance corrected for prism constant and PPM
- SD_3** Slope distance corrected for prism constant, PPM, instrument and target heights
- HD_1** Horizontal distance between instrument point and target point
- VD_1** Vertical distance between instrument point and target point
- H_I** Height of instrument

H_T Height of target

PC Prism constant

NOTE – Interim corrections are not shown in the diagram. They are enclosed in square brackets.

Prism constant correction

The prism constant is applied to all slope distances. It is usually negative, but can be positive.

$$SD_1 = SD_R + PC$$

where:

SD_R measured (raw) slope distance

SD_1 resultant slope distance

PC prism constant

PPM correction

The parts per million (PPM) correction is applied to the slope distance after being corrected for the prism constant (see above). PPM depends on pressure and temperature.

$$D_2(P, T) = SD_1 \left[J - \frac{N \cdot P}{273.16 + T} \right] \cdot 10^{-6}$$

where:

P air pressure in millibars

T temperature in °C

J & N constants supplied by the EDM manufacturer

The following table lists some manufacturers of conventional instruments, and the **J** (RefractiveIndex) and **N** (CarrierWavelength) constants that the Trimble Access software uses to compute the PPM correction for those instruments.

Manufacturer of conventional instruments	J constant	N constant
Trimble VX/S/M Series	from instrument	from instrument
Trimble 5600	274.41	79.39
Trimble 3300/3600	278.77	80.653
Trimble TTS300/500	270.0	79.167
Sokkia SET	279.0	79.400

Manufacturer of conventional instruments	J constant	N constant
Topcon	279.7	79.600
Geotronics 400/600	275.0	79.550
Leica	282.0	79.400
Zeiss Elta2/Elta3/Elta4	255.0	79.100
Zeiss Elta C	281.8	79.391
Pentax	279.0	79.400
Nikon	275.0	79.5065

NOTE –

- The Trimble VX/S/M Series instrument **J** and **N** constants that are read from the instrument can be viewed in an exported JobXML file.
- The **J** constant value is the refractive index of the instrument. The **N'** constant value is for use with pressure measurements in millibars. In the .dc file, the value is converted to one that can be used with pressure measurements in mmHg.

Curvature and refraction correction

The curvature and refraction correction is applied to vertical angles according to the coefficient of refraction that you set.

$$VA_3 = VA_2 - \left[\frac{(CO_{nOff} - k \times RO_{nOff}) \times SD_1}{2R} \right] \times \frac{180}{\pi}$$

where:

- CO_{nOff}** if the *Curvature correction* option is selected, this value is 1; otherwise it is 0
- RO_{nOff}** if the *Refraction correction* option is selected, this value is 1; otherwise it is 0
- k** coefficient of terrestrial refraction, specified in the **Refraction const.** field in the **Corrections** screen
- R** approximate spheroid radius = 6378137m. (WGS-84 semi-major axis)
- SD₁** slope distance, from equation - [Prism constant correction](#)
- VA₂** vertical angle, from instrument
- VA₃** corrected vertical angle

Instrument and target height reduction

The corrected vertical angle (VA_4) from the instrument to the target is:

$$VA_4 = \tan^{-1} \left[\frac{SD_2 \sin VA_3}{SD_2 \cos VA_3 + H_I - H_T} \right]$$

where:

H_I instrument height

H_T target height

SD_2 slope distance

VA_3 vertical angle, from equation - [Curvature and refraction correction](#)

VA_4 corrected vertical angle

The slope distance from source point to target point (SD_3) is given by the following:

$$SD_3 = \frac{SD_2 \sin VA_3}{\sin VA_4} \quad (\text{Point to point slope distance})$$

Face 1/Face 2 determination

This section describes how the Trimble Access software reduces Face 2 readings to Face 1 readings in order to perform calculations. It does this automatically.

The observed raw vertical angle is used to determine whether an observation is Face 1 or Face 2:

- If the vertical angle is not present the observation is assumed to be Face 1.
- If the vertical angle is in the range 0° to 180° , the observation is Face 1.
- If the vertical angle is in the range 180° - 360° , the observation is Face 2.

Orientation correction

To orient circle readings so that they become azimuths, an orientation correction is applied. The orientation correction is the difference between the backsight circle reading and the backsight azimuth. This term is applied to all the other observations (circle readings) at a station.

The formula is:

$$Az_x = HA_x + (Az_B - HA_B) \quad (\text{Orientation correction})$$

where:

Az_x azimuth to any point X

HA_x horizontal observation to any point X

Az_B actual backsight azimuth ("reference azimuth")

HA_B observed backsight circle reading

Slope reduction

The horizontal and vertical components of an observation (**HD_1** and **VD_1**) are found from the vertical angle and slope distances by:

$$HD_1 = SD_3 \sin VA_4$$

$$VD_1 = SD_3 \cos VA_4$$

where:

HD_1 horizontal distance

VD_1 vertical distance

VA_4 zenith angle

SD_3 slope distance

Coordinate calculation

Coordinates of a target point are calculated from observations and the coordinates of the instrument point using:

$$N_2 = N_1 + HD_1 \cos Az_1$$

$$E_2 = E_1 + HD_1 \sin Az_1$$

$$Z_2 = Z_1 + VD_1$$

where:

N_1, E_1, Z_1 Northing, Easting, Elevation of instrument point

N_2, E_2, Z_2 Northing, Easting, Elevation of target point

HD_1 horizontal distance

VD_1 vertical distance

Az_1 from equation - [Orientation correction](#).

Mean turned angle calculations

When the software calculates the mean turned angle and the mean distance for a point, it also calculates standard errors as follows:

For the angles, the standard error of the mean of a set of measurements formula is used:

$$\sigma = \sqrt{\frac{\Sigma v^2}{n(n-1)}}$$

For distances, the standard error of the set of measurements formula is used:

$$\sigma = \sqrt{\frac{\Sigma v^2}{(n-1)}}$$

Resection calculation

The resection calculation is a least-squares calculation that uses all available data.

Observations to the same point taken on different faces are treated as separate observations. However, the results are the same as those gained from using meaned (averaged) observations.

The residuals are given for each *point*, not each observation.

The formula used for the standard errors is as follows:

$$\sigma = \frac{(\sqrt{\Sigma v^2})}{(n-1)}$$

Traverse calculation

This section shows the formulae that the software uses when it calculates a traverse.

Compass adjustment

The Compass adjustment distributes the error in proportion to the length of the traverse lines. The formulae are as follows:

$$\text{Northing adjustment} = \frac{D}{\Sigma D} \times \text{Northing misclosure}$$

where:

D horizontal distance

ΣD sum of the horizontal distances in the traverse

$$\text{Easting adjustment} = \frac{D}{\Sigma D} \times \text{Easting misclosure}$$

where:

D horizontal distance

ΣD sum of the horizontal distances in the traverse

Transit adjustment

The Transit adjustment distributes the error in proportion to the northing and easting of each traverse point.

$$\text{Northing adjustment} = \frac{\Delta N}{\Sigma \Delta N} \times \text{Northing misclosure}$$

where:

ΔN change in northing for the traverse line

$\Sigma \Delta N$ sum of the changes in northings of all the traverse lines

$$\text{Easting adjustment} = \frac{\Delta E}{\Sigma \Delta E} \times \text{Easting misclosure}$$

where:

ΔE change in easting for the traverse line

$\Sigma \Delta E$ sum of the changes in eastings of all the traverse lines

Angular adjustments

There are three options for distributing the angular error in a traverse:

- Proportional to distance—the error is distributed among the angles based on the sum of the inverses of the forward and back traverse distances for each point. The formula used is as follows:

$$A_a = \frac{\frac{1}{\text{to dist}} + \frac{1}{\text{from dist}}}{\Sigma \left(\frac{1}{\text{to dist}} + \frac{1}{\text{from dist}} \right)} \times A_m$$

where:

A_a Angular adjustment

A_m Angular misclosure

- Equal proportions – the error is distributed evenly among the angles in the traverse.
- None – the error is not distributed.

Elevation adjustments

There are three options for distributing the elevation error in a traverse:

- Proportional to distance – the error is distributed in proportion to the length of the traverse line to the point.
- Equal proportions – the error is distributed evenly among the traverse lines.
- None – the error is not distributed.

Standard errors recorded with conventional observations

Each conventional observation that is recorded in the job file has standard errors associated with it. The standard errors that are recorded with the observations are determined as follows:

- For a single observation to a point (for example, Measure topo) the standard error values assigned to the observation are the instrument standard error values (a-priori values). This is because there is nothing else that the standard errors can be determined from.
If a distance fails to reach the instrument's rated accuracy (typically due to an unstable target), Trimble Access saves the achieved standard deviation of the measurement. When this occurs, a message tells you that the instrument's distance standard deviation was not achieved for this observation.
- Computed 'observations' for example, Dual prism offsets, Distance offsets, Circular Object and Remote Object measurement methods, the standard errors are recorded as null.

Area calculations

Grid areas

When the *Distances* field is set to Grid, the area computed is the area at sea level and is calculated using grid coordinates. Elevations are not used in the computation.

Ellipsoid areas

When the *Distances* field is set to Ellipsoid, the ground area is computed by multiplying the grid area by the projection scale factor (at the centroid of the area) squared.

Ground areas

When the *Distances* field is set to Ground, the ground area is computed by multiplying the grid area by the combined scale factor (at the centroid of the area) squared.

The combined scale factor (at the centroid of the area) is the point scale factor multiplied by the sea level factor,

where, the point scale factor is computed for the centroid of the area based on the current projection definition, and the sea level factor is:

$$\left(\frac{\bar{h} + R}{R} \right)$$

So the ground area is:

$$GA \times \left(PSF \times \left(\frac{\bar{h} + R}{R} \right) \right)^2$$

where:

$$\bar{h} = \frac{\sum h_i}{N}$$

<i>GA</i>	Grid area
<i>PSF</i>	Point scale factor
<i>N</i>	number of elements with elevations
\bar{h}	average elevation
<i>R</i>	ellipsoidal radius